Kempe equivalence of 3-edge-colorings in cubic graphs

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Let G be a cubic graph having a 3-edge-coloring. For a 2-edge-colored cycle D, a Kempe switch (at D) is an operation to obtain another 3-edge-coloring by switching the colors of E(D). Two 3-edge-colorings in G are Kempe equivalent if one is obtained from the other by the sequence of Kempe switches. Kempe equivalence is a well-known method to study cubic graphs, such as the 4 Color Theorem.

Fisk proved that every bipartite planar graph has only one Kempe equivalence class, and Mohar asked which cubic graphs have such a property. In this talk, we consider the projective planar case, showing that a bipartite cubic graph G on the projective-plane admits only one Kempe equivalent class if and only if the dual G^* is not 4-vertex-colorable. In order to prove this theorem, we use the *signature* of 3-edge-colorings.