On the existence of path-induced-saturated graphs

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Let G and H be graphs. We say that G is an *H-saturated graph* if G has no subgraph isomorphic to H, but adding any non-edge to G creates a subgraph isomorphic to H. We can always construct an n-vertex H-saturated graph for all H and $n \geq |V(H)|$. Historically, the maximum and minimum number of edges among n-vertex H-saturated graphs have been intensively studied.

A trigraph $T = (V(T), E_B(T), E_W(T), E_G(T))$ consists of the vertex set $V(T)$, the black edge set $E_B(T)$, the white edge set $E_W(T)$, and the gray edge set $E_G(T)$. A realization of T is a graph $G = (V(G), E(G))$ such that $V(G) = V(T)$ and $E(G) = E_B(T) \cup S$ for some $S \subseteq E_G(T)$. As an induced analogue of an H-saturated graph, we say that a trigraph T is *H-induced-saturated* if no realization of T has an induced subgraph isomorphic H, but adding any element of $E_B(T) \cup E_W(T)$ to $E_G(T)$ creates an induced subgraph isomorphic to H in some realization of T. It follows that a graph G is *H-induced-saturated* if G has no induced subgraph isomorphic to H, but adding any non-edge to G or deleting any edge from G creates an induced subgraph isomorphic to H. Unlike an H-saturated graph, an H-induced-saturated graph does not always exist. In fact, in 2012, Martin and Smith showed that P_4-induced-saturated graphs do not exist, where we use P_n to denote a path on n vertices. In 2019, Axenovich and Csikós asked the existence of P_n-induced-saturated graphs for $n \geq 5$; it is easy to construct such graphs when $n \in \{2, 3\}$. Recently, Räty constructed a graph that is P_6-induced-saturated.

In this talk, we show some examples of P_n-induced-saturated graphs for $n \geq 6$. In particular, we show a family of P_{3m}-induced-saturated graphs, where m is a positive integer at least 2. This talk is based on joint works with Ilkyoo Choi and Boram Park.