Degeneracy and Colorings of Squares of Planar Graphs without 4-Cycles

Ilkyoo Choi[1]

[1]*Hankuk University of Foreign Studies*

August 6, 2019

We prove several results on coloring squares of planar graphs without 4-cycles. First, we show that if G is such a graph, then G^2 is $(\Delta(G)+72)$-degenerate. This implies an upper bound of $\Delta(G)+73$ on the chromatic number of G^2 as well as on several variants of the chromatic number such as the list-chromatic number, paint number, Alon–Tarsi number, and correspondence chromatic number. We also show that if $\Delta(G)$ is sufficiently large, then the upper bounds on each of these parameters of G^2 can all be lowered to $\Delta(G)+2$ (which is best possible). To complement these results, we show that 4-cycles are unique in having this property. Specifically, let S be a finite list of positive integers, with $4 \notin S$. For each constant C, we construct a planar graph $G_{S,C}$ with no cycle with length in S, but for which $\chi(G_{S,C}^2) > \Delta(G_{S,C}) + C$.