IMPROPER COLORINGS OF SPARSE GRAPHS ON SURFACES

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A graph is $(d_1, \dots, d_r)$-colorable if its vertex set can be partitioned into r sets $V_1, \dots, V_r$ where the maximum degree of the graph induced by V_i is at most d_i for each $i \in \{1, \dots r\}$. Given r and $d_1, \dots, d_r$, determining if a (sparse) graph is $(d_1, \dots, d_r)$-colorable has attracted much interest. For example, the Four Color Theorem states that all planar graphs are 4-colorable, and therefore $(0, 0, 0, 0)$-colorable. The question is also well studied for partitioning planar graphs into three parts. For two parts, it is known that for given d_1 and d_2, there exists a planar graph that is not (d_1, d_2)-colorable. Therefore, it is natural to study the question for planar graphs with girth conditions. Namely, given g and d_1, determine the minimum $d_2 = d_2(g, d_1)$ such that planar graphs with girth g are (d_1, d_2)-colorable.

We continue the study and ask the same question for graphs that are embeddable on a fixed surface. Given integers k, γ, g we completely characterize the smallest k-tuple $(d_1, \dots, d_k)$ in lexicographical order such that each graph of girth at least $g \leq 7$ that is embeddable on a surface of Euler genus γ is $(d_1, \dots, d_k)$-colorable. All of our results are tight, up to a constant multiplicative factor for the degrees d_i depending on g. In particular, we show that a graph embeddable on a surface of Euler genus γ is $(0, 0, 0, K_1(\gamma))$-colorable and $(2, 2, K_2(\gamma))$-colorable, where $K_1(\gamma)$ and $K_2(\gamma)$ are linear functions in γ.

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